Nonlinear large amplitude vibration of lattice core CNTRC cylindrical panels resting on elastic foundation

Dang Thuy Dong, Vu Minh Duc, Tran Quang Minh, Nguyen Van Tien, Nguyen Thi Phuong, Vu Hoai Nam*
University of Transport Technology, Ha Noi, Vietnam
hoainam.vu@utt.edu.vn

Abstract

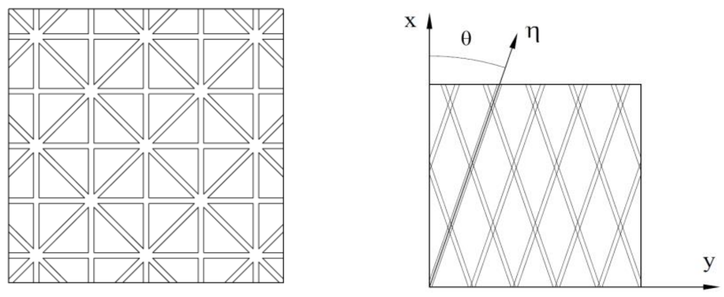
In this paper, a new design of sandwich cylindrical panels is proposed with two functionally graded carbon nanotube reinforced composite (FG-CNTRC) face sheets and lattice core. The lattice core layer is modeled by improving the novel smeared technique for stiffener according to the first-order shear deformation theory (FSDT). Nonlinear vibration behavior of first-order shear deformable cylindrical panels with the geometric nonlinearities is analyzed in present paper. The stress function is considered and the Galerkin method is used to formulate the nonlinear motion equation system. Nonlinear dynamic responses of panels can be achieved by using the fourth order Runge-Kutta method. Numerical investigations can show the very large effects of lattice core layer, volume fraction of carbon nanotube, type of carbon nanotube distribution on the nonlinear vibration behavior of sandwich FG-CNTRC cylindrical panels.

Introduction

With the excellent thermomechanical and the physical characteristics, the FG-CNTRC structures are widely used in the construction, aviation, aerospace, etc… [3,10,11]. The vibration characteristics are important factors in the design stages. Therefore, recently, some scientists have focused on the vibration analysis of the plate and shell. Basing on the Mori-Tanaka approach, Formica et al. [6] considered the vibration behavior of CNTRC plates by applying an equivalent continuum mod-el. By using the perturbation technique, Shen and Xiang [9] presented a nonlinear vibration analysis of FG-CNTRC cylindrical panel in thermal environments. Ansari et al. [1, 2] investigated the linear free vibration of FG-CNTRC elliptical plates [2] and spherical shells [1] by a numerical approach based on the FSDT. Kiani et al. [8] considered the free vibration of FG-CNTRC skew cylindrical shells by applying the Chebyshev-Ritz formulation. Chakraborty et al. [4] presented a semi-analytical approach to analyze the vibration of FG-CNTRC cylindrical shell panels. Duc et al. [5] investigated the fundamental frequencies and dynamic responses of FG-CNTRC doubly curved shells under thermal loading. Based on the Donnell shell theory, Foroutan et al. [7] investigated the nonlinear vibration of imperfect FG-CNTRC cylindrical panels, which were subjected to external pressure and nonlinear temperature distribution in the thickness direction.

In this work, a new design of sandwich structures is proposed with two FG-CNTRC face sheets and lattice core. The lattice core layer is modeled by improving the novel smeared technique for stiffener according to the FSDT. The nonlinear dynamic analysis of FG-CNTRC panels with lattice core and shell-foundation interaction is presented. The fundamental frequency and dynamic response of shell are derived by employing Galerkin method and Runge-Kutta method. Effects of lattice core layer, geometrical and material parameters on the nonlinear vibration behaviors of cylindrical panel are investigated. The obtained results of this paper provide the significant algorithm and experience for the engineering designs of these structures.

Methodology



a. Lattice stiffeners system

b. Local coordinate system of oblique stiffeners

Fig. 1. Configuration of a lattice core FG-CNTRC cylindrical panels

The volume fractions of the CNTs corresponding to CNT distributions types are presented as

$$V_{CNT}=\begin{cases} V_{CNT}^{*} & (UD) \\ 2V_{CNT}^{*}\left(1-2|z|/h\right) & (FG-O) \\ 4V_{CNT}^{*}\ 2|z|/h & (FG-X) \end{cases} \quad (1)$$

Basing on the FSDT, the strain components include the strains are defined as in Ref. [1,2]. The deformation compatibility equation is expressed as in Ref. [7]. Hooke's law is employed for FG-CNTRC layers of cylindrical panel as

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{bmatrix}=\begin{bmatrix} Q_{11}^{i} & Q_{12}^{i} & 0 & 0 & 0 \\ Q_{12}^{i} & Q_{22}^{i} & 0 & 0 & 0 \\ 0 & 0 & Q_{66}^{i} & 0 & 0 \\ 0 & 0 & 0 & Q_{44}^{i} & 0 \\ 0 & 0 & 0 & 0 & Q_{55}^{i} \end{bmatrix}\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix}, \quad (2)$$

By applying the homogenization technique for the lattice core layer, the expressions of internal forces are obtained

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}=\begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} \end{bmatrix}\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \phi_{x,x} \\ \phi_{y,y} \\ \phi_{x,y}+\phi_{y,x} \end{Bmatrix} \quad (5)$$

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix}=K_s\begin{Bmatrix} H_{44}\gamma_{xz} \\ H_{55}\gamma_{yz} \end{Bmatrix}=K_s\begin{Bmatrix} H_{44}w_{,x}+H_{44}\phi_x \\ H_{55}w_{,y}+H_{55}\phi_y \end{Bmatrix}, \quad (6)$$

Basing on the FSDT, the nonlinear motion equations of panel are written as [1,2]

$$\begin{aligned} &N_{xy,y}+N_{x,x}=I_1u_{,tt}+I_2\phi_{x,tt}, \\ &N_{y,y}+N_{xy,x}=I_1v_{,tt}+I_2\phi_{y,tt}, \\ &Q_{y,y}+Q_{x,x}+N_x\left(w_{,xx}+w_{,xx}^{*}\right)+N_y\left(w_{,yy}+w_{,yy}^{*}\right)+2N_{xy}\left(w_{,xy}+w_{,xy}^{*}\right)-K_1w+K_2\nabla^2w+q+N_y/R=I_1w_{,tt}+2\varepsilon I_1w_{,t}, \\ &M_{xy,y}+M_{x,x}-Q_x=I_2u_{,tt}+I_3\phi_{x,tt}, \\ &M_{y,y}+M_{xy,x}-Q_y=I_2v_{,tt}+I_3\phi_{y,tt}, \end{aligned} \quad (14)$$

From the deformation compatibility equation, the stress function is determined. Substituting the obtained stress function and solutions from Eq. (15) into motion Eqs. (14) and then applying Galerkin method

$$\begin{aligned} &h_{11}W+h_{12}\Phi_x+h_{13}\Phi_y+h_{14}(W+\xi h)\Phi_x+h_{15}(W+\xi h)\Phi_y+c_1(W+\xi h) \\ &+e_1(W+2\xi h)(W+\xi h)W+e_2Q\sin\Omega t=I_1d^2W/dt^2+2\varepsilon I_1dW/dt, \\ &h_{21}W+h_{22}\Phi_x+h_{23}\Phi_y+e_4W(W+2\xi h)=\hat{I}_3d^2\Phi_x/dt^2, \\ &h_{31}W+h_{32}\Phi_x+h_{33}\Phi_y+e_5W(W+2\xi h)=\hat{I}_3d^2\Phi_y/dt^2, \end{aligned} \quad (17)$$

From Eq. (17), the fundamental frequency of lattice core CNTRC cylindrical panels is determined by

$$\begin{vmatrix} h_{11}+I_1\omega^2 & h_{12} & h_{13} \\ h_{21} & h_{22}+\hat{I}_3\omega^2 & h_{23} \\ h_{31} & h_{32} & h_{33}+\hat{I}_3\omega^2 \end{vmatrix}=0. \quad (18)$$

Results

Table 1. Comparisons of parameters of fundamental frequencies $\bar{\omega}=\omega\sqrt{\rho_m/E_m}\left(b^2/h\right)$ of FG-CNTRC cylindrical panels

V_{CNT}^{*}	UD		FG- X	
	Shen and Xiang [9]	Present	Shen and Xiang [9]	Present
0.12	18.5407	18.8906	22.0781	22.6793
0.17	23.0831	23.7054	27.3541	27.8963
0.28	26.5256	26.9001	32.1718	32.7039

Table 2. Effects of the lattice core thickness on the fundamental frequency of lattice core CNTRC cylindrical panel (×10³ rad/s, θ=45°)

CNT distribution types	h_{core}/h , (m, n) = (1, 1)				
	0.6	0.5	0.4	0.3	0.2
UD	2.4680	2.5032	2.5383	2.5735	2.6091
FG- O	2.4669	2.5013	2.5356	2.5700	2.6047
FG- X	2.4712	2.5077	2.5442	2.5808	2.6177

Table 3. Effects of angle θ on the fundamental frequency of lattice core CNTRC cylindrical panel (×10³ rad/s), $h_{core}=0.3h$

CNT distribution types	θ , (m, n) = (1, 1)						
	$\pi/12$	$\pi/9$	$\pi/6$	$\pi/4$	$\pi/3$	$7\pi/18$	$5\pi/12$
UD	2.5249	2.5350	2.5558	2.5735	2.5635	2.5451	2.5354
FG- O	2.5213	2.5314	2.5523	2.5700	2.5599	2.5414	2.5317
FG- X	2.5322	2.5423	2.5631	2.5808	2.5707	2.5523	2.5427

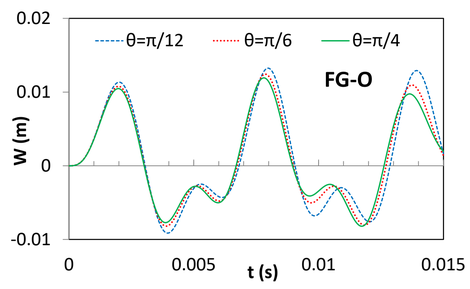


Fig. 2. Effect of angle θ on the dynamic response curves of lattice core CNTRC cylindrical panel (q=10⁵sin (1000t) N/m²).

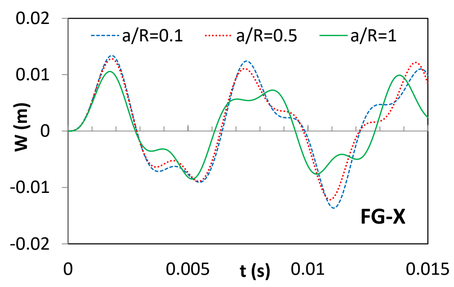


Fig. 3. Effect of a/R ratio on the dynamic response curves of lattice core CNTRC cylindrical panel (q=10⁵sin (1000t) N/m²), θ=π/4

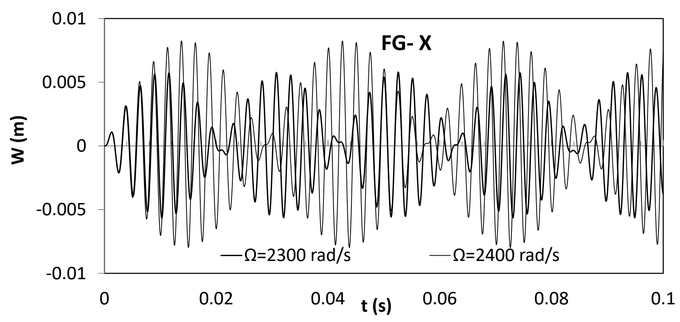


Fig. 4. Harmonic beat phenomenon of lattice core CNTRC cylindrical panel (q=10⁴ sin(Ωt) N/m²), θ=π/4

Conclusion

Governing formulations of nonlinear vibration of lattice core CNTRC cylindrical panel based on the FSDT are established. By employing the Galerkin method, the nonlinear dynamic equations and the expression of fundamental frequency are achieved. By applying the Runge-Kutta numerical approximation, the dynamic response curves are investigated. Effects of lattice core, geometrical and material parameters on the nonlinear vibration of shells are considered in numerical results.

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